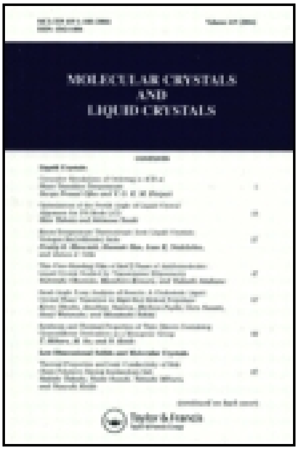

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On the Cutoff Frequency in the Conduction Mode Electroydrodynamic Instability of a Nematic Layer

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The dependence of the cut-off frequency of the conduction mode electrohydrodynamic instabilities. exhibited by homogeneously aligned negative dielectric anisotropy nematic liquid crystals, on the conductivity of the nematic material, the thickness of the nematic layer and the intensity of an orienting magnetic field, is examined. It is found that for sufficiently high conductivity and/or large layer thickness, the cut-off frequency is proportional to the conductivity of the material, and independent of the intensity of the orienting field. Lowering the conductivity and/or the thickness of the nematic layer results in smaller values of the ratio of the cut-off frequency to conductivity, until a critical limit is reached beyond of which no conduction mode of instability has to be observed. It is found also that for given values of conductivity and thickness, the effect of the orienting magnetic field is to lower the cut-off frequency of the instability and, also, to increase the critical conductivity and thickness values.

Keywords: Liquid Crystal; Nematic; Electrohydrodynamics

1.INTRODUCTION.

The electrohydrodynamic instabilities (EHDI) exhibited by (mainly negative dielectric anisotropy and homogeneously aligned) nematic liquid crystal (NLC) layers[1-4] are manifested in two modes. Namely, the conduction mode (CM) and the dielectric mode (DM) of EHDI. The former is characterized by lower threshold voltages and, also, by smaller distortion wave-numbers than the latter. In both modes the threshold voltage of the EHDI, and its distortion wavenumber, are increasing functions of the frequency of the applied AC voltage, except in a relatively narrow frequency range (sometimes called the transition region) where the CM, beyond of its threshold, is characterized also by an upper voltage limit, which is smaller than the threshold voltage of the DM of EHDI at the same frequency. This situation is depicted in figure 1. As the figure suggests, the difference between the threshold and the upper limit voltages decreases with increasing frequency, and becomes zero at a certain frequency f_c, the cut-off frequency (COF) of the CM, beyond of which no CM of EHDI is observed. To the contrary, for the DM of EHDI there exits no experimental evidence for such a cut-off behavior, as far as its frequency dependence is considered.

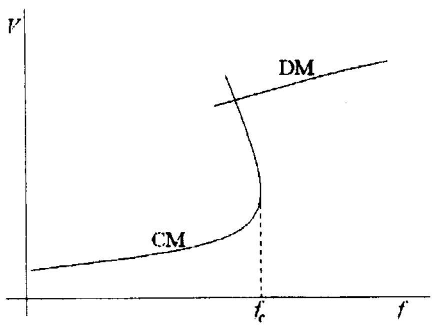

FIGURE 1. Frequency dependence of the voltage thresholds.

The physical reason behind this experimental behavior is as follows. In the CM of EHDI the space charge density of the NLC layer oscillates around zero, following the alternations of the applied AC voltage, whereas the director field oscillates, with small amplitude, around a non-zero mean distortion. So, when the period of the applied voltage is sufficiently larger than the relaxation time of the space charge of the NLC layer, the latter has enough time to follow the applied electric field, and thus the CM is observable. It is obvious that at sufficiently large frequencies, i.e. small periods of the applied AC voltage, the situation reverses and thus the manifestation of the CM of EHDI is impossible.

In the DM of EHDI, on the other hand, the roles of the space charge and the director field are reversed. Furthermore, the relaxation time of the director field, which in this case alternates with the applied AC electric field, is a decreasing function of the applied electric field. So, for any frequency value, it is possible to decrease sufficiently the director relaxation time, by increasing the intensity of the electric field across the layer, permitting thus its oscillations around zero, according to the alternations of the applied voltage. This is why the DM of EHDI is not characterized by any frequency cutoff.

The relaxation time of the space charge is proportional to the conductivity of the nematic material. So, the COF of the CM of the EHDI should be an increasing function of the conductivity. This fact, which has been verified experimentally, is explained also theoretically by an approximate calculation[3,4] according to which there exists a proportionality relation between the COF and the conductivity of the nematic material. In this article we show that this relation is accurate to the extent that the conductivity of the nematic material and/or the thickness of the nematic layer are sufficiently large. We also show, by numerical means, that this proportionality ceases to be valid for lower conductivity and thickness values, and that as the conductivity of the NLC and/or the thickness of the nematic layer decreases so does the ratio of the COF to the conductivity. Furthermore, the calculations show that, for a given layer thickness, there exists a critical value of the conductivity such that no CM of EHDI has to be observed when the conductivity of the nematic material is lesser than this critical value. The same is true for the thickness of the nematic layer of a given conductivity. Finally, the effect of an aligning magnetic field is examined. We conclude that, for a nematic layer of a given conductivity and thickness, the effect of this field is to decrease the COF. Also in the presence of such a stabilizing field the already mentioned critical values of the conductivity and of the thickness of the layer have larger values.

The calculations are based on the linearized two-dimensional theory of the EHDI of nematic layers of reference [5,6]. Aside from some simplifying assumptions, on which it is based, this theory is more amenable to calculations, such that we need for the problem in hand, than the more rigorous formulations of the EHDI theory[7-11], published more recently. These formulations are too involved to be able to lead to results analogous to those exposed in the foregoing. So we prefer to have results and to leave the question of their degree of accuracy to a future numerical research.

2. FORMULATION.

We consider a negative dielectric anisotropy NLC layer of thickness L (figure 2) homogeneously aligned between two conductive planes. Let x be the axis defined by the orientation direction of the undisturbed layer, parallel to the layer boundaries, and z the axis normal to the layer. An AC voltage $V(t)$ applied across the layer excites it electrohydrodynamically, provided its amplitude exceeds a threshold value, which, in general, depends on the frequency of the applied AC voltage as well as on its waveform.

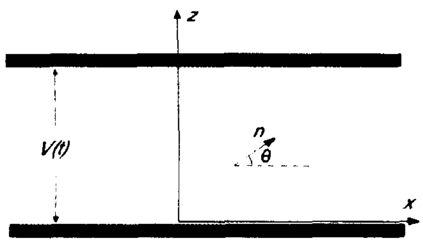

FIGURE 2. System layout.

The nature of the problem we consider here forces us to define and use the reduced time $\tau \equiv \sigma_{\parallel} t$ and the reduced frequency $f_r \equiv f/\sigma_{\parallel}$ instead of the time t and the frequency f of the applied voltage, where $\sigma_{\parallel}$ is the conductivity of the NLC parallel to its director. Now, according to the theory of reference [5], and after some redefinitions of the parameters involved, the differential equations governing the dynamical behavior of the NLC layer, under near-threshold conditions, are

$$\frac{dr}{d\tau} + (a_1 + \lambda a_D)r + b_1 V_r \psi = 0, \quad \frac{d\psi}{d\tau} + \lambda a_2 \psi + \lambda b_2 V_r r = 0 \qquad (1,a,b)$$

In these equations $\psi \equiv \partial\theta/\partial x$, where θ is the angle between the director and the x-axis, $r \equiv \rho L/\pi\sqrt{K_3}$ where ρ is the space charge density in the NLC layer and K_3 is the bend elastic constant of the NLC, $V_r \equiv V/\pi\sqrt{K_3}$ is the reduced voltage across the NLC layer and

$$\lambda \equiv K_3\pi^2/\sigma_{\parallel}L^2 \tag{2}$$

a parameter with the dimensions of viscosity coefficient, which has a central role in our calculations. Furthermore

$$a_1 \equiv \frac{4\pi\xi_\sigma}{\varepsilon_{\parallel}\xi_\varepsilon} \qquad a_D \equiv D_r\xi_D \qquad b_1 \equiv \left(\frac{\varepsilon_\perp}{\varepsilon_\parallel} - \frac{\sigma_\perp}{\sigma_\parallel}\right)\frac{\xi}{\xi_\varepsilon}$$

$$a_2 \equiv p_0 - p_1V_r^2 \qquad p_0 \equiv \frac{\xi_K + Q_B}{H} \qquad p_1 \equiv \frac{1}{H}\frac{\varepsilon_\alpha\varepsilon_\perp}{4\pi\varepsilon_\parallel}\frac{\xi}{\xi_\varepsilon}$$

$$b_2 \equiv \frac{S^2}{H}\left(\frac{\xi_\alpha}{\xi_\eta} - \frac{\varepsilon_\alpha}{\varepsilon_\parallel\xi_\varepsilon}\right)$$

with $S \equiv qL/\pi$ the distortion ratio of the layer, where q is the distortion wavenumber of the electrohydrodynamically excited NLC layer.

$$Q_B \equiv \chi_a B^2L^2/K_3\pi^2 \tag{3}$$

where B is the intensity of the orienting magnetic field, χ_α is the anisotropy of the magnetic susceptibility of the NLC. In addition

$$\xi \equiv S^2 + 1, \qquad \xi_\sigma \equiv S^2 + \sigma_\perp/\sigma_\parallel, \qquad \xi_\varepsilon \equiv S^2 + \varepsilon_\perp/\varepsilon_\parallel,$$

$$\xi_D \equiv S^2 + D_\parallel/D_\perp, \qquad \xi_K \equiv S^2 + K_1/K_3$$

where $\sigma_\perp$ is the conductivity in a direction perpendicular to the nematic director, $\varepsilon_\parallel$, $\varepsilon_\perp$ are the dielectric constants in the direction parallel an perpendicular to the director, respectively, $\varepsilon_\alpha \equiv \varepsilon_\parallel - \varepsilon_\perp$ the dielectric anisotropy of the NLC supposed to be negative, $D_\parallel$, $D_\perp$ are the diffusion coefficients of the NLC in the direction parallel an perpendicular to the director, respectively, $D_r \equiv D_{\|}/K_3$ and K_1 is the splay elastic constant of the NLC. Finally, in terms of the five independent viscosity coefficients $\alpha_1,\ldots,\alpha_5$ of the incompressible nematic phase we define

$$\xi_a \equiv \alpha_3 - \alpha_2 S^2, \qquad \xi_\eta \equiv \eta_1 S^4 + \eta_3 S^2 + \eta_2$$

and
$$H \equiv \alpha_3 - \alpha_2 - \xi_a^2/\xi_\eta$$

where

$$\eta_1 \equiv (\alpha_4 + \alpha_5 - \alpha_2)/2 \qquad \eta_2 \equiv \eta_1 + \alpha_3 + \alpha_2, \qquad \eta_3 \equiv \alpha_1 + \alpha_3 + \alpha_4 + \alpha_5$$

Now, assuming a symmetric square pulse of amplitude V_r as the waveform of the applied AC voltage, the condition for the system of equations (1) to has a non-zero solution is

$$\sinh\left(\frac{a_1 + \lambda(a_2 + a_D)}{4f_r}\right) = u\frac{a_1 - \lambda(a_2 - a_D)}{D}\sinh\left(\frac{D}{4f_r}\right) \tag{4}$$

where
$$D \equiv \left\{[a_1 - \lambda(a_2 - a_D)]^2 + 4\lambda b_1 b_2 V_r^2\right\}^{1/2}$$

In equation (4), $u = +1$ refers to the CM whereas $u = -1$ to the DM of EHDI. So, setting $u = +1$ equation (4) gives us the implicit dependence of the amplitude V_r of the applied square-pulsed AC voltage on the distortion ratio S of the EHDI and on the reduced frequency f_r of the applied voltage. Thus, for a given value of the reduced frequency, the minimum and maximum values of V_r, considered as a function of S, are the threshold and the upper voltages, respectively, for the occurrence of the CM of EHDI, while the respective values of S define the distortion wavenumbers of the NLC layer under these conditions. As figure 1 suggests, and the calculations confirm, the difference of these extrema of V_r decreases with increasing f_r, tending to zero when the reduced frequency tends to a definite value f_{rc}, which is the reduced COF of the CM of the EHDI, for the values of the material parameters used in the calculations.

3. RESULTS.

Among the parameters defined in the previous section, λ and Q_B which are defined by equations (2) and (3), respectively, control the effects of the conductivity, the layer thickness and the intensity of the ori-

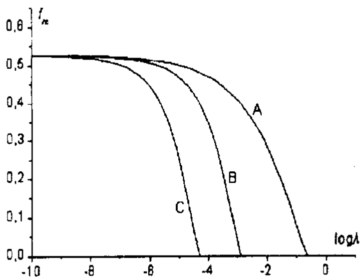

FIGURE 3. Cutoff frequency versus λ for different values of Q_B.

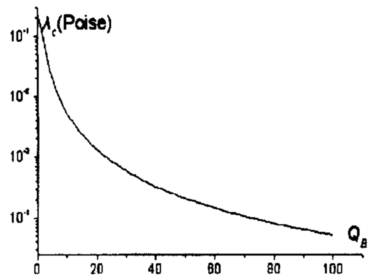

FIGURE 4. Depedence of the critical λ on Q_B.

enting field, on the CM of the EHDI and, especially, on the COF. Using equation (4), and following the lines of thought sketched in the previous section, we computed the dependence of the reduced COF as a function of λ, for three different values of Q_B. The material parameters used for the calculation are those of the MBBA room temperature nematic, and they are given in an appendix at the end of the article. The results of the calculations are plotted in figure 3, where curves A, B and C refer to $Q_B = 0$, 20 and 100[12]. The main conclusions that can be drawn from these plots are as follows: a) For sufficiently small values of the parameter (that is for sufficiently highly conducting and/or sufficiently thick layer), the reduced COF is nearly independent of λ as well as the intensity of the orienting field. Thus, under these conditions, the COF is essentially proportional to the conductivity of the NLC material, and the proportionality constant does not depend on the intensity of the orienting magnetic field. This fact will be shown analytically as well in the next section. b) For any value of the parameter Q_B, the reduced COF gets smaller when increasing λ. Thus for a NLC layer with moderate conductivity and thickness values the COF ceases to be proportional to the conductivity, being a more sensitive function of it. Furthermore, there exists a critical value λ_c of the parameter λ such that for any layer with λ greater than λ_c no CM of EHDI has to expected to take place. This critical value is a very sensitive decreasing function of the orienting magnetic field. Figure 4 depicts the dependence of λ_c on Q_B, as is calculated for the case of the nematic MBBA.

4. COF FOR A LAYER WITH HIGH CONDUCTIVITY AND/OR LARGE THICKNESS.

Assuming that the conductivity and/or the thickness of the NLC layer, which is driven by the square pulsed AC voltage, are sufficiently large so that the parameter λ, defined by equation (3), has a sufficiently small value, we expand the two sides of equation (4) in powers of λ around the value $\lambda = 0$, and retain terms up to first order in λ. Solving, then, the resulting equation with respect to the reduced amplitude of the applied AC voltage, and using the parameter definitions of section 2, we get the equation

$$V_r^2 = p_0 \Bigg/ \left[\frac{b_1 b_2}{a_1} \left(1 - \frac{4f_r}{a_1} \tanh\left(\frac{a_1}{4f_r} \right) \right) + p_1 \right] \tag{5}$$

which gives the reduced amplitude of the applied AC voltage as a function of its reduced frequency and of the distortion ratio. The minimum value of V_r, considered as a function of S, is the reduced threshold voltage, and the corresponding value of S defines the wavenumber of the distortion of the NLC. In order for these to work, the denominator of the right hand side of equation (5) should be positive. So we must have

$$\frac{4f_r}{a_1} \tanh\left(\frac{a_1}{4f_r} \right) < 1 + \frac{p_1 a_1}{b_1 b_2} \tag{6}$$

The function $x \tanh(1/x)$ is a monotonically increasing function of x. Thus for any value of S, and for the given material parameters, there exists a number γ such that the inequality (6) reduces to $4f_r/a_1 < \gamma$. As one can easily test, for a nematic with negative dielectric anisotropy, the right hand side of (6), as well as a_1, are increasing functions of S. Consequently the same is true for γ. Thus, using the definitions of section 2, and letting $S \to \infty$, the condition (6) for the CM of EHDI to be observable, becomes

$$\frac{\varepsilon_{\parallel} f_r}{\pi} \tanh\left(\frac{\pi}{\varepsilon_{\parallel} f_r}\right) < 1 - \frac{\varepsilon_a / \varepsilon_{\perp} \varepsilon_{\parallel}}{\left(\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}} - \frac{\sigma_{\perp}}{\sigma_{\parallel}}\right)\left(\frac{\alpha_2}{\eta_1} + \frac{\varepsilon_a}{\varepsilon_{\parallel}}\right)}$$

which is an implicit relation giving us the reduced COF in the case of a square pulsed excitation. For the nematic MBBA the right hand side of the last relation is equal to 0.6745. Thus solving the inequality numerically we have, for the MBBA nematic, $f_r < 0.527$ and, thus, $f < 0.527\sigma_{\parallel}$. The value 0.527 is exactly the same with the one we got, in the limit where $\lambda \to 0$, when calculating the curves of figure 3.

Following a similar reasoning we can compute the reduced COF in the limit of small values of λ, for the case of a sinusoidal excitation. To do so we consider the variables ψ and r to be expanded as power series of λ

$$\psi = \sum_{n=0} \psi_n \lambda^n \quad \text{and} \quad r = \sum_{n=0} r_n \lambda^n \tag{7}$$

Inserting these expressions in equations (1), and equating the coefficients of equal orders in λ we have

$$\frac{dr_0}{d\tau} + a_1 r_0 + b_1 V_r \psi_0 = 0 \qquad \frac{d\psi_0}{d\tau} = 0$$

$$\frac{dr_n}{d\tau} + a_1 r_n + b_1 V_r \psi_n = a_D r_{n-1} \qquad \frac{d\psi_n}{d\tau} + a_2 \psi_{n-1} + b_2 V_r r_{n-1} = 0$$

$n = 1,2,..$ (8,a,b)

Assuming a sinusoidal voltage of the form $V_r = V_{rm} \sin \omega_r \tau$, where ω_r is its reduced angular frequency, the first two equations give

$$\psi_0 = \text{constant} \qquad\qquad r_0 = (\omega_r \sin\omega_r\tau - a_1\cos\omega_r\tau)/(a_1^2 + \omega_r^2)$$

In the second equation, and in order to have a steady solution, the arbitrary coefficient of the exponential term is chosen to be zero. Inserting r_0 to equation (8b) with $n = 1$ we find

$$\psi_1 = \psi_0\left(\frac{b_1 b_2 a_1 V_{rms}^2}{a_1^2 + \omega_r^2} - p_0 + p_1 V_{rms}^2\right)\tau + C_1\cos\omega_r\tau + C_2\sin\omega_r\tau \tag{9}$$

where C_1 and C_2 are quantities independent of τ. According to equation (8a), the steady solution for the r_1 is a sum sines and cosines of $\omega_r\tau$ and $3\omega_r\tau$. We assume, now, that the value of the parameter λ is sufficiently small so that the expansions (7) can be stopped at the first order terms. So the quantities ψ_0, ψ_1, r_0 and r_1 describe satisfactorily the dynamical behavior of the NLC layer. But, according to equation (9), in order to have a steady dynamical behavior of the layer, the coefficient in parentheses should be zero. Thus we have the equation

$$V_{rms}^2 = p_0 \bigg/ \left(\frac{b_1 b_2 a_1}{a_1^2 + \omega_r^2} + p_1\right) \tag{10}$$

which is the analog of the equation (5) for the case of sinusoidal excitation. Equation (10) makes sense provided the denominator of the right hand side is a positive quantity. This happens when

$$\omega_r^2 < -a_1^2(1 + b_1 b_2/a_1 p_1)$$

For a NLC with $\varepsilon_a < 0$ the quantity at the right hand side of the last inequality is positive and an increasing function of the distortion ratio S. Thus letting $S \to \infty$, and using the parameter definitions of section 2, we get, finally, the relation

$$f_r < \frac{2}{\varepsilon_\parallel}\left\{\left(1 - \frac{\varepsilon_\parallel}{\varepsilon_\perp}\frac{\sigma_\perp}{\sigma_\parallel}\right)\left(\frac{\alpha_2}{\eta_1}\frac{\varepsilon_\parallel}{\varepsilon_a}\right) - 1\right\}^{1/2}$$

which gives the reduced COF of the CM, in the case of sinusoidal excitation. In the case of the MBBA nematic, at 25°C, this relation gives

$f_c = 0.6386\sigma_{\|}$, which is larger than the COF frequency for a square pulse excitation, a fact that is already known by experiment. The last relation has already been derived in the literature with different reasoning[3,4].

APPENDIX.

The material parameters used for the calculations, which characterize the room temperature and negative dielectric anisotropy nematic MBBA at 250C, are as follows.

Elastic constants[13]: $K_3 = 8.61 \times 10^{-7}$ dynes, $K_1 = 6.66 \times 10^{-7}$ dynes.

Viscosity coefficients[14] : $\alpha_1 = 0.065$ Poise, $\alpha_2 = -0.775$ Poise, $\alpha_3 = -0.012$ Poise, $\alpha_4 = 0.832$ Poise, $\alpha_5 = 0.463$ Poise

Electrical properties[4] : $\varepsilon_{\|} = 4.72$, $\varepsilon_{\perp} = 5.25$, $\sigma_{\|}/\sigma_{\perp} = D_{\|}/D_{\perp} = 1.5$, $D_{\|} = 5 \times 10^{-8}$ cm^2 sec^{-1}.

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